

**Unit 3: Powers and Exponents**

| Day | Topic | Assignment |
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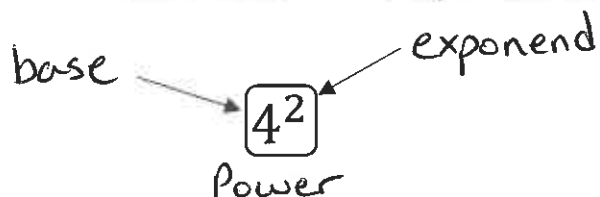
**Test Day:** \_\_\_\_\_

**3.1 – Using Exponents to Describe Numbers**

**Exponential Form:** A shorter way of writing repeated multiplication, using a base and an exponent.

- $5 \times 5$  in exponential form is  $5^2$ .
- The above is read as “5 to the power of 2” or “5 squared”.

**Power:** An expression made up of a base and an exponent.



**Base:** The number in a power that you multiply by itself.

**Exponent:** Tells you the number of times that the base is multiplied by itself.

- In  $4^3$ , 4 is multiplied by itself 3 times:  $4^3 = \underline{4 \times 4 \times 4}$ .

**Example 1:** Write as a Power in Exponential Form then Evaluate

a)  $2 \times 2 \times 2 \times 2$       4 factors of 2.

$$= 2^4$$

$$= 16$$

b)  $(-3)(-3)(-3)(-3)$       4 factors of  $(-3)$

$$= (-3)^4$$

$$= 81$$

c)  $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$       3 factors of  $\left(\frac{1}{2}\right)$

$$= \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

d)  $n \times n \times n \times n \times n$       5 factors of  $n$ .

$$= n^5$$

\*\*Be careful with **NEGATIVE** bases, notice the difference between:

$$\begin{array}{l} \text{base} = 4 \\ \text{exponent} = 2 \end{array} \quad \begin{array}{l} -4^2 \\ = -(4^2) \\ = -(4 \times 4) \\ = -16 \end{array}$$

$$\begin{array}{l} (-4)^2 \\ \text{base} = (-4) \\ \text{exponent} = 2 \end{array} \quad \begin{array}{l} = (-4) \times (-4) \\ = 16. \end{array}$$

**Example 2:** Evaluate each Power

a)  $9^2$

$$\begin{aligned} &= 9 \times 9 \\ &= 81 \end{aligned}$$

b)  $-3^2$

$$\begin{aligned} &= -(3^2) \\ &= -(3 \times 3) = -9 \end{aligned}$$

c)  $(-2)^4$

$$\begin{aligned} &= (-2) \times (-2) \times (-2) \times (-2) \\ &= 4 \times 4 = 16 \end{aligned}$$

d)  $(-5)^3$

$$\begin{aligned} &= (-5) \times (-5) \times (-5) \\ &= (25) \times (-5) = -125 \end{aligned}$$

e)  $\left(\frac{1}{3}\right)^3$

$$\begin{aligned} &= \left(\frac{1}{3}\right) \times \left(\frac{1}{3}\right) \times \left(\frac{1}{3}\right) \\ &= \frac{1}{27} \end{aligned}$$

f)  $-1^{100}$

$$\begin{aligned} &= -(1^{100}) \\ &= -1 \end{aligned}$$

g)  $2(-3)^2$

$$\begin{aligned} &= 2 \times (-3) \times (-3) \\ &= 2 \times 9 = 18 \end{aligned}$$

Practice: Textbook pg. 79/#

1-3, 5-7, 9, 12, 16

**3.2 – Exponent Laws****Example 1: Multiplying Powers with the Same Base****Rule:** When multiplying powers with the same base, we can **ADD** the exponents.

$$a^m \times a^n = a^{m+n}$$

a)  $2^3 \times 2^2$

$= 2^{3+2}$

$= 2^5$

$$\left\{ \begin{array}{l} 2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2) \\ = 2^5 \end{array} \right.$$

**Example 2: Dividing Powers with the Same Base****Rule:** When dividing powers with the same base, we can **SUBTRACT** the exponents.

$$a^m \div a^n = a^{m-n}$$

a)  $5^8 \div 5^3$

$= 5^{8-3}$

$= 5^5$

$$\left\{ \begin{array}{l} 5^8 \div 5^3 = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5} \\ = 5^5 \end{array} \right.$$

**Example 3: Raising a Power to Another Power****Rule:** When a power is raised to another power, we can **MULTIPLY** the exponents.

$$(a^m)^n = a^{mn}$$

a)  $(2^3)^2$

$= 2^{3 \cdot 2}$

$= 2^6$

$$\left\{ \begin{array}{l} (2^3)^2 = (2^3) \times (2^3) \\ = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \\ = 2^6 \end{array} \right.$$

**Give it a Try:** Use Exponent Rules to Write as a Single Power. Do not Evaluate

a)  $4^3 \times 4^5$

$= 4^{3+5}$

$= 4^8$

b)  $\left(\frac{1}{3}\right)^5 \left(\frac{1}{3}\right)^6$

$= \left(\frac{1}{3}\right)^{5+6}$

$= \left(\frac{1}{3}\right)^{11}$

c)  $8^5 \div 8^4$

$= 8^{5-4}$

$= 8^1$

d)  $\frac{(-4)^3}{(-4)^2}$

$= (-4)^{3-2}$

$= (-4)^1$

e)  $(3^2)^4$

$= 3^{2 \cdot 4}$

$= 3^8$

f)  $(y)^3 \times (y)^5$

$= y^{3+5}$

$= y^8$

**Give it a Try:** Use Exponent Rules to Write as a Single Power, then Evaluate

a)  $(2)^2(2)^3(2)^1$

$= (2)^{2+3+1}$

$= (2)^6$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

b)  $\left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^4$

$= \left(\frac{1}{2}\right)^{2+4}$

$= \left(\frac{1}{2}\right)^6 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{64}$

c)  $12^6 \div 12^5$

$= 12^{6-5}$

$= 12$

d)  $\frac{(6)^{10}}{(6)^8}$

$= (6)^{10-8}$

$= (6)^2$

$= 6 \times 6$

$= 36$

$$\begin{aligned}
 \text{e) } & ((-3)^2)^2 \\
 &= (-3)^{2 \cdot 2} \\
 &= (-3)^4 \\
 &= (-3) \times (-3) \times (-3) \times (-3) \\
 &= 81
 \end{aligned}$$

$$\begin{aligned}
 \text{f) } & \frac{x^5}{x^3} \\
 &= x^{5-3} \\
 &= x^2
 \end{aligned}$$

**Example 4: Raising a Product to a Power**

**Rule:** When a product is raised to a power, we can distribute the power to each factor in the product.

$$(ab)^m = a^m \times b^m$$

$$\begin{aligned}
 \text{a) } & (3 \times 2)^2 \\
 &= 3^2 \times 2^2 \\
 &= 9 \times 4 \\
 &= 36
 \end{aligned}$$

$$\begin{aligned}
 (3 \times 2)^2 &= (3 \times 2) \times (3 \times 2) \\
 &= 3^2 \times 2^2
 \end{aligned}$$

**Example 5: Raising a Quotient to a Power**

**Rule:** When a quotient is raised to a power, we can distribute the power to both the numerator and the denominator of the quotient.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$\begin{aligned}
 \text{a) } & \left(\frac{2}{3}\right)^3 \\
 &= \frac{2^3}{3^3} \\
 &= \frac{8}{27}
 \end{aligned}$$

$$\begin{aligned}
 \left(\frac{2}{3}\right)^3 &= \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \\
 &= \frac{2 \times 2 \times 2}{3 \times 3 \times 3} \\
 &= \frac{2^3}{3^3}
 \end{aligned}$$

**Example 6:** Bases with Exponents of Zero

$$* \frac{a^n}{a^n} = a^{n-n} = a^0 = 1$$

**Rule:** All numbers (except zero) raised to the power of zero equal one.

$$a^0 = 1$$

**Give it a Try:** Use Exponent Rules to Simplify, then Evaluate

$$\begin{aligned} \text{a) } (2 \times 3)^4 &= 2^4 \times 3^4 \\ &= 16 \times 81 \\ &= 1296 \end{aligned}$$

$$\begin{array}{r} 4 \\ 16 \\ \times 81 \\ \hline 1280 \\ 1296 \end{array}$$

$$\begin{aligned} \text{b) } (4 \times 5)^2 &= 4^2 \times 5^2 \\ &= 16 \times 25 \\ &= 400 \end{aligned}$$

$$\begin{array}{r} 31 \\ 16 \\ \times 25 \\ \hline 180 \\ 320 \\ \hline 400 \end{array}$$

$$\begin{aligned} \text{c) } \left(\frac{1}{3}\right)^2 &= \frac{1^2}{3^2} \\ &= \frac{1}{9} \end{aligned}$$

$$\begin{aligned} \text{d) } \left(\frac{2}{5}\right)^3 &= \frac{2^3}{5^3} \\ &= \frac{8}{125} \end{aligned}$$

$$\begin{aligned} \text{e) } (-8)^0 &= 1 \end{aligned}$$

$$\begin{aligned} \text{f) } (5n)^2 &= 5^2 n^2 \\ &= 25n^2 \end{aligned}$$

Practice: Textbook pg. 86/# 1-3, 5-9, 11, 14, 15, 20

| Exponent Laws              |                                                |
|----------------------------|------------------------------------------------|
| <b>Multiplying Powers</b>  | $a^m \times a^n = a^{m+n}$                     |
| <b>Dividing Powers</b>     | $a^m \div a^n = a^{m-n}$                       |
| <b>Power of a Power</b>    | $(a^m)^n = a^{mn}$                             |
| <b>Power of a Product</b>  | $(ab)^m = a^m \times b^m$                      |
| <b>Power of a Quotient</b> | $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ |
| <b>Zero Exponent</b>       | $a^0 = 1$                                      |

**3.3 – Order of Operations with Exponents**

**Coefficient:** A number that multiplies an expression.

- In  $-5(4)^2$ , the coefficient is -5.

**Example 1:** Evaluate Powers with Coefficients

a)  $3(2)^4$

$$= 3 \times 2^4$$

$$= 3 \times 2 \times 2 \times 2 \times 2$$

$$= 48$$

c)  $-4^4$

$$= -[4^4]$$

$$= -[4 \times 4 \times 4 \times 4]$$

$$= -256$$

**Example 2:** Evaluate Expressions with Powers

a)  $4^2 - 8 \div 2 + (-3^2)$

$$= 16 - 8 \div 2 + (-9)$$

$$= 16 - 4 - 9$$

$$= 16 - 13$$

$$= 3$$

c)  $8(5 + 2)^2 - 12 \div 2^2$

$$= 8(7)^2 - 12 \div 4$$

$$= 8(49) - 3$$

$$= 392 - 3 = 389$$

b)  $-3(-5)^2$   $(-5)^2 = (-5)(-5)$

$$= -3(25)$$

$$= -75$$

d)  $6(7)^0$   $(7)^0 = 1$

$$= 6(1)$$

$$= 6$$

**BEDMAS!**

b)  $-2(-15 - 4^2) + 4(2 + 3)^3$

$$= -2(-15 - 16) + 4(5)^3$$

$$= -2(-31) + 4(125)$$

$$= 62 + 500$$

$$= 562$$

d)  $-[2 + 3(2^3 + 4^2)]$

$$= -[2 + 3(8 + 16)]$$

$$= -[2 + 3(24)]$$

$$= -[2 + 72]$$

Brackets  
Exponents  
Multiplication  
Division  
Addition  
Subtraction.

Practice: Textbook pg. 91/#

$$1, 2, 4 \text{ ace}, 5 \text{ ace}, 6, 15, 16 = -74$$

3.4 – Using Exponents to Solve Problems**Example 1:** Use Formulas to Solve Problems

Write an expression using exponents to solve each problem.

- a) What is the surface area of a cube with an edge length of 4cm?

The surface area of a cube is

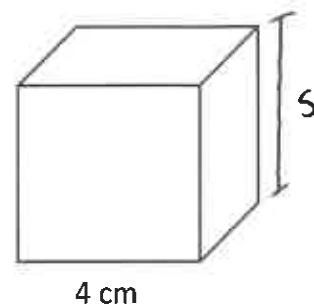
$$SA = 6s^2$$

$$\text{If } s = 4$$

$$SA = 6(4)^2$$

$$= 6(16)$$

$$= 96$$

The surface area is  $96 \text{ cm}^2$ 

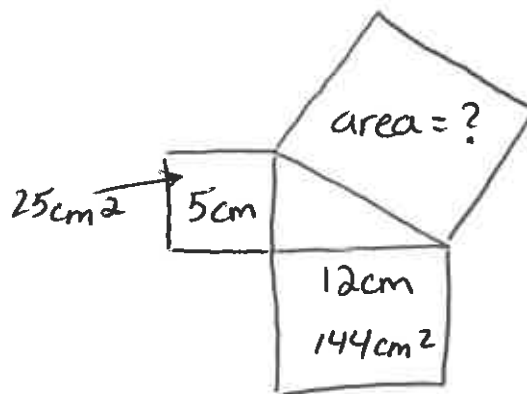
- b) Three squares are attached to a right triangle. Find the area of the square attached to the hypotenuse in the diagram.

$$a^2 + b^2 = c^2$$

$$(5)^2 + (12)^2 = c^2$$

$$25 + 144 = c^2$$

$$169 = c^2$$

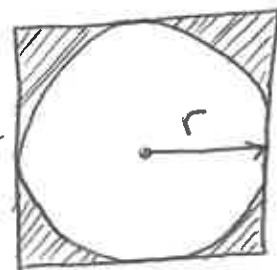
The area of the square attached to the hypotenuse is  $169 \text{ cm}^2$ .

- c) A circle is inscribed in a square with a side length of 20cm. What is the area of the shaded region?

$$\text{Area of shaded region} = \text{Area of square} - \text{Area of circle}$$

$$\begin{aligned} A_{\text{square}} &= s^2 \\ &= 20^2 \\ &= 400 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{circle}} &= \pi r^2 \\ &= \pi (10)^2 \\ &= \pi (100) \\ &\approx 314 \text{ cm}^2 \end{aligned}$$



20 cm

$$r = \frac{d}{2}$$

$$r = \frac{20 \text{ cm}}{2}$$

$$r = 10 \text{ cm}$$

$$A_{\text{shaded region}} \approx 400 - 314 = \boxed{86 \text{ cm}^2}$$

**Example 2:** Develop a Formula to Solve a Problem

A dish holds 100 bacteria. Under ideal conditions, the bacteria double in number every hour. How many bacteria will be present after each number of hours?

- a) 1 hour
- b) 5 hours
- c) n hours

a) After 1h, the number doubles.

$$100 \times 2 = 200 \text{ bacteria.}$$

b) After 5h, the number doubles five times.

$$100 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= 100(2^5) = 100(32) = 3200 \text{ bacteria.}$$

c) After n hours, the number doubles n times.

$$\text{Number of bacteria} = 100(2^n)$$

Practice: Textbook pg. 98/# 1-3, 5, 10