4.1 – Introduction to Polynomials

Algebraic Expression: A mathematical phrase that combines numbers and variables connected by mathematical operations.

Polynomial: An expression with one or more terms separated by addition or subtraction.

- **Monomial**: A polynomial with one term, for example $-3x$.
- **Binomial**: A polynomial with two terms, for example $5a + 7b$.
- **Trinomial**: A polynomial with three terms, for example $2m^2 + 3m - 8$.

Variable: A symbol used to represent an unknown number or quantity.

Term: A single number, or variable, or an expression formed by the product of numbers and variables.

Coefficient: The number part of a term that is multiplied by the variable.

Constant: A term with no variable.

Degree:

- **Degree (of a term)**: The sum of the exponents on the variables in a single term.
- **Degree (of a polynomial)**: The value of the highest-degree term in a polynomial.

Example 1: Identify Coefficients and Variables

In the table below, identify the coefficient and variable(s) for each expression.

Expression	Coefficient	Variable(s)
$25x$	25	x
$-4.9t^2$	-4.9	t
lw	1	l, w
2.5	2.5	none.

Example 2: Identify the Degree of a Term

For each term in the table, state the degree.

4x → degree = 1

Term	Degree
$5x^2$	2
$-4xy^2$	3
a^3bc^2	6
4	0

degree = 1 + 2 = 3

degree = 3 + 1 + 2 = 6

Example 3: Classify Polynomials

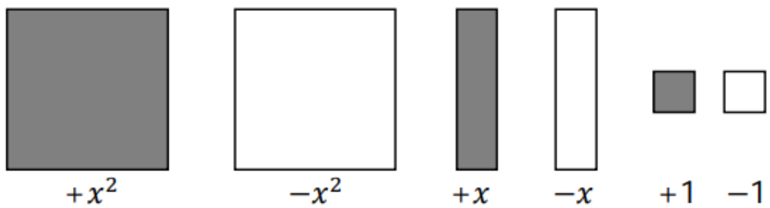
Classify each polynomial by the number of terms, polynomial type, and degree.

Expression	Number of Terms	Polynomial Type	Degree
$5w^2$	1	monomial	2
$6y - 2$	2	binomial	1
$3x^2 - 9x^1y^1 + y^2$	3	trinomial	2
$m^1 + m^1n^1 - 3n + 5$	4	four-term polynomial	2
124	1	monomial	0

Math 9 Name: Key

Algebra Tiles

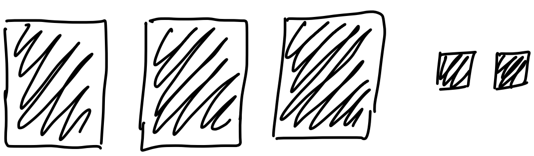
We can use algebra tiles to model algebraic expressions.



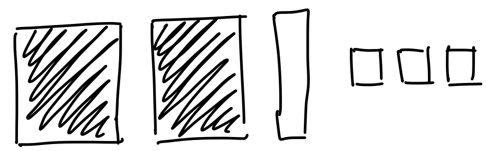
* Textbook uses colours for +'ve tiles and white for -'ve tiles.

Example 4: Model Each Expression with Algebra Tiles

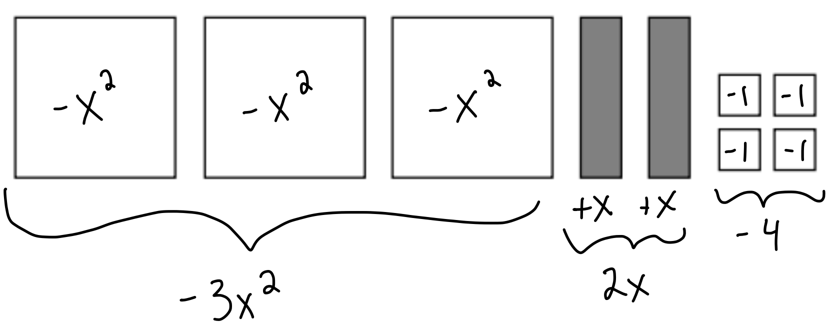
$3x^2 + 2$



$2x^2 - x - 3$



Example 6: Identify the expression from Algebra Tiles



$\boxed{-3x^2 + 2x - 4}$

Practice: Textbook pg. 112/# 1acef, 2-7, 9, 10, 14, 16

 Name: Key.

4.2 – Adding and Subtracting Polynomials

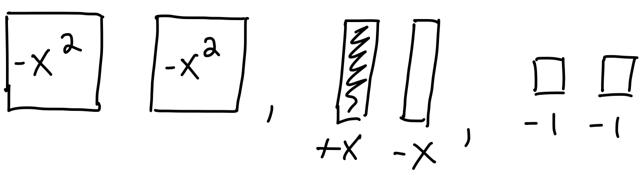
Zero Pairs: Two tiles that represent opposite values.

- The sum of the two tiles is 0.

examples $+1$ and -1, $+x$ and $-x$, $+x^2$ and $-x^2$

Like Terms: Terms that have the same variable(s) and the same exponents on the variable(s).

- Constant terms are "like".
- With algebra tiles, like terms have the same shape and size.



Example 1: Identifying Like Terms

Identify and collect the like terms for the following polynomials.

a) $-3x + \boxed{2x^2} + x - 4 + \boxed{x^2} + 5$

b) $\boxed{4x^2} - 3x + 1 - 3 + \boxed{2x^2} + x$

$2x^2 + x^2 - 3x + x + 4 + 5$

$= 3x^2 - 2x + 9.$

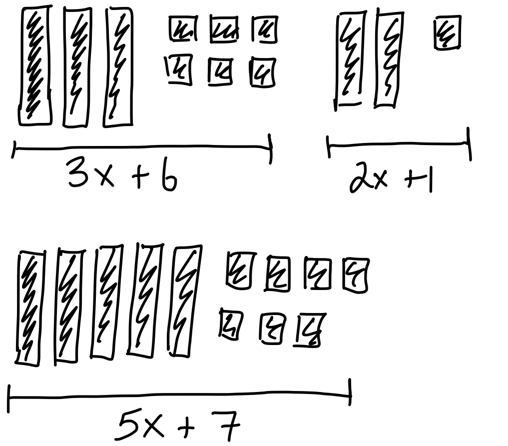
$4x^2 + 2x^2 - 3x + x + 1 - 3$

$= 6x^2 - 2x - 2.$

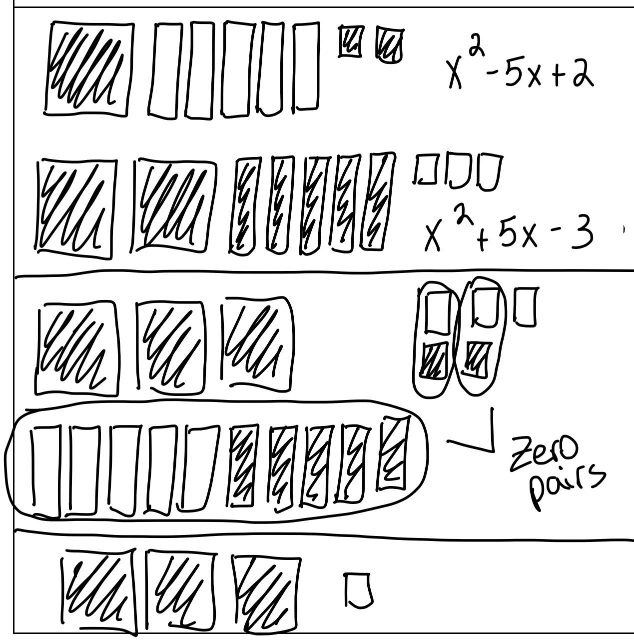
Example 2: Add Polynomials

What is the sum of each pair of polynomials?

a) $3x + 6$ and $2x + 1$

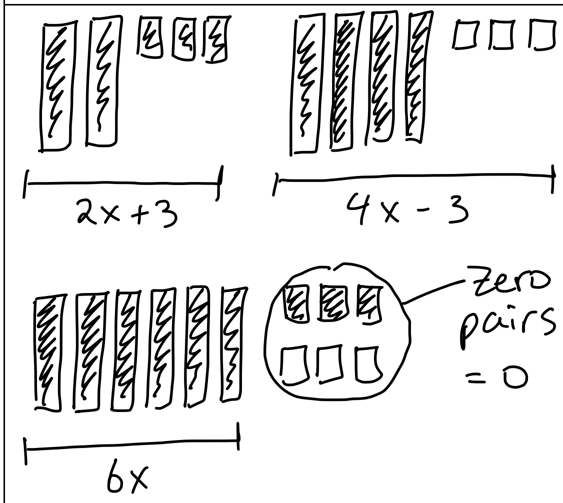
Method 1: Use a model	Method 2: Solve algebraically
 $(3x+6) + (2x+1) = 5x + 7.$	$(3x+6) + (2x+1)$ $= 3x + 2x + 6 + 1$ * put like terms together. $= 5x + 7$ * Combine like terms.

b) $x^2 - 5x + 2$ and $2x^2 + 5x - 3$

Method 1: Use a model	Method 2: Solve algebraically
	$(x^2 - 5x + 2) + (2x^2 + 5x - 3)$ $= x^2 + 2x^2 \underbrace{-5x + 5x}_{=0} + 2 - 3$ $= 3x^2 - 1$

$= 3x^2 - 1$

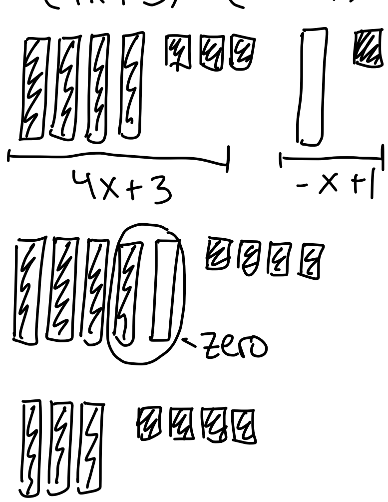
c) $(2x+3)$ and $(4x-3)$

Method 1: Use a model	Method 2: Solve algebraically
 $(2x+3)+(4x-3) = 6x$	$(2x+3)+(4x-3)$ $= 2x+4x+\cancel{3}-\cancel{3}$ $= 2x+4x$ $= 6x.$

Example 3: Subtract Polynomials

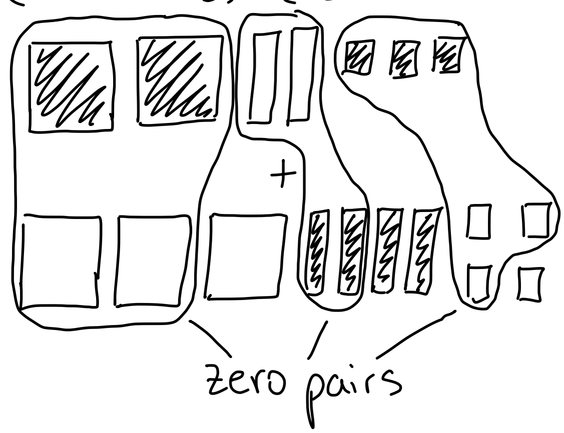
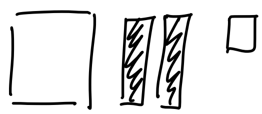
Simplify the following algebraic expressions.

a) $(4x+3)-(x-1)$

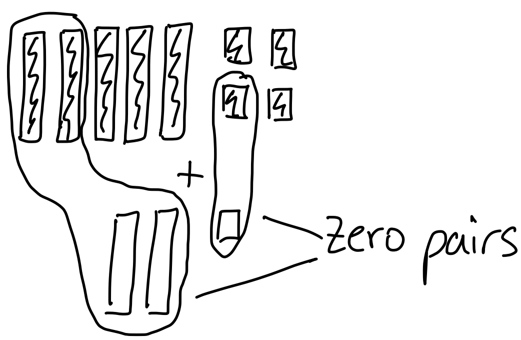
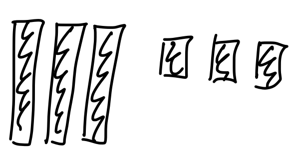
Method 1: Add the opposite using algebra tiles	Method 2: Solve algebraically
$(4x+3)+(-x+1)$ 	$(4x+3)+(-x+1)$ $= 4x-x+3+1$ $= 3x+4$

$(4x+3)-(x-1) = 3x+4$

b) $(2x^2 - 2x + 3) - (3x^2 - 4x + 4)$

Method 1: Add the opposite using algebra tiles	Method 2: Solve algebraically
$(2x^2 - 2x + 3) + (-3x^2 + 4x - 4)$  $=$  $= -x^2 + 2x - 1$	$(2x^2 - 2x + 3) + (-3x^2 + 4x - 4)$ $= 2x^2 - 3x^2 - 2x + 4x + 3 - 4$ $= -x^2 + 2x - 1$

c) $(5x + 4) - (2x + 1)$

Method 1: Add the opposite using algebra tiles	Method 2: Solve algebraically
$(5x + 4) + (-2x - 1)$  $=$  $= 3x + 3$	$(5x + 4) + (-2x - 1)$ $= 5x - 2x + 4 - 1$ $= 3x + 3$

 Name: Key

Example 4: Model and Solve Problems with Polynomials

The table shows the costs involved to rent a banquet hall.

Charge Type	Fixed Cost ($)	Cost Per Person ($)
Banquet Hall	1500	0
Service Charges	0	5
Food Costs	500	45
Drink Costs	100	20

a) What is the cost to hold a banquet for 200 people?

$C = 1500 + 5(200) + 500 + 45(200) + 100 + 20(200)$

$= 1500 + 1000 + 500 + 9000 + 100 + 4000$

$= \$16,100$

b) Write an expression for each charge type for *n* people.

let n = # of people.

Banquet Hall: 1500

Food Costs: $500 + 45n$

Service Charges: $5n$

Drink Costs: $100 + 20n$

c) Write an expression to rent the banquet hall for *n* people. Simplify the polynomial.

$C = 1500 + 5n + 500 + 45n + 100 + 20n$

$= 2100 + 70n$

d) Verify that your expression works by using it to determine the cost of a banquet for 200 people.

$C(200) = 2100 + 70(200)$

$= 2100 + 14,000$

$= 16,100$

Practice: Textbook pg. 123/# 1, 3, 4, 6, 7ace, 9, 11, 13

Math 9 Name: Key

4.3 – Multiplying and Dividing Monomials

Multiplying Monomials

Recall: Exponent laws.

$$3^2 \times 3^4 = 3^{2+4} = 3^6$$

Now, if the bases were a variable we have,

$$x^2 \times x^4 = x^{2+4} = x^6$$

To multiply two monomials:

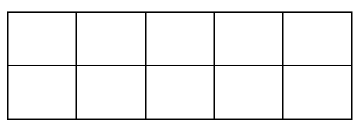
- Multiply the coefficients
- Multiply the variables, using exponent laws

When we use a model, we can think of multiplying monomials as finding the area of a rectangle.

$$Area = length \times width$$

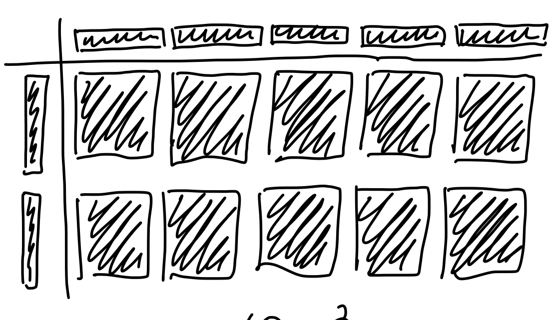
5 cm

2 cm

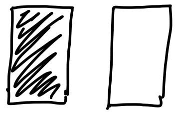


Example 1: Multiply each pair of monomials.

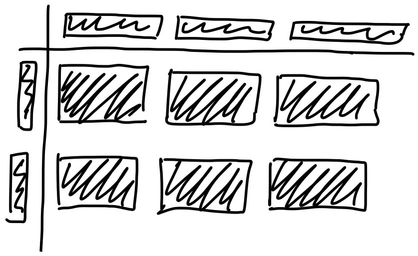
a) $(5x)(2x)$

Method 1: Use a model	Method 2: Solve algebraically
 $= 10x^2$	$(5x)(2x)$ $5 \cdot 2 \cdot x \cdot x$ $= 10x^2$

Math 9 Name: Key

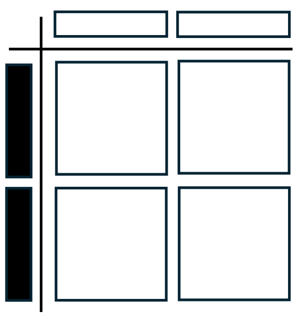
$+xy$ $-xy$  width y, length x.

b) $(3x)(2y)$

Method 1: Use a model	Method 2: Solve algebraically
 $= 6xy$	$(3x)(2y)$ $= 3 \cdot 2 \cdot x \cdot y$ $= 6xy$

Example 2: Write a monomial multiplication statement for each set of algebra tiles.

a)



$(2x)(-2x) = -4x^2$

b)

$(-2x)(3x) = -6x^2$

Example3: Determine the product of each pair of monomials.

a) $(-4x)(2x)$

$= -4 \cdot 2 \cdot x \cdot x$

$= -8x^2$

b) $(3y)(7y)$

$= 3 \cdot 7 \cdot y \cdot y$

$= 21y^2$

c) $(5x)(-3y)$

$= 5 \cdot (-3) \cdot x \cdot y$

$= -15xy$

d) $(6m)(-0.2m)$

$= 6 \cdot (-0.2) \cdot m \cdot m$

$= -1.2mn$

e) $(\frac{2}{3}n)(12n)$

$= \left(\frac{2}{3}\right) \cdot 12 \cdot n \cdot n \quad = \frac{2}{\cancel{3}} \times \frac{\overset{4}{\cancel{12}}}{1} n^2 = 8n^2$

Dividing Monomials

Recall: Exponent laws

$$\frac{2^5}{2^2} = 2^{5-2} = 2^3$$

Now, if the bases were a variable we have,

$$\frac{x^5}{x^2} = x^{5-2} = x^3$$

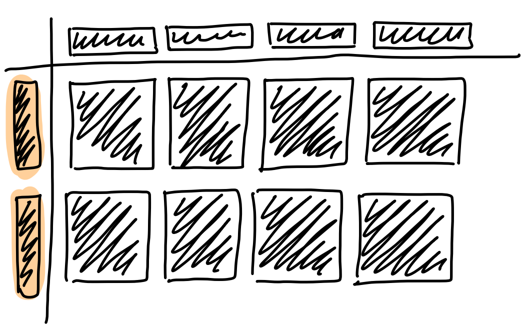
To divide two monomials:

- Divide the coefficients
- Divide the variables, using exponent laws

When we use a model, we can think as if we know the area of the rectangle and one of the side lengths and we need to find the missing side length by dividing.

Example 4: Divide each pair of monomials.

a) $(8x^2) \div (4x)$

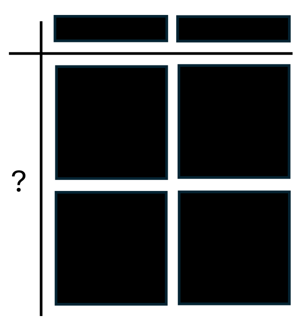
Method 1: Use a model	Method 2: Solve algebraically
 $= 2x$	$(8x^2) \div (4x)$ $= (8 \div 4)(x^2 \div x)$ $= 2x$

b) $\frac{-4xy}{2y}$

Method 1: Use a model	Method 2: Solve algebraically
 $= -2x$	$\frac{-\cancel{4}x\cancel{y}}{\cancel{2}\cancel{y}}$ $= -2x$

Example 5: Write a monomial division statement for each set of algebra tiles.

a)



$4x^2 \div 2x = 2x$

b)

$-6x^2 \div 3x = -2x$

Example 6: Determine the quotient of each pair of algebra tiles.

a) $\frac{\overset{-2}{\cancel{16}}x^2}{-\cancel{8}x}$

$= -2x$

b) $\frac{\overset{5}{\cancel{15}}x\cancel{y}}{\cancel{3}\cancel{y}}$

$= 5x$

c) $\frac{\cancel{-9}\cancel{m}\cancel{n}}{\cancel{-3}\cancel{m}\cancel{n}}$

$= 3$

d) $\frac{12\cancel{x}y}{8\cancel{x}}$

$= \frac{\overset{3}{\cancel{12}}y}{\underset{2}{\cancel{8}}}$

$= \frac{3y}{2}$

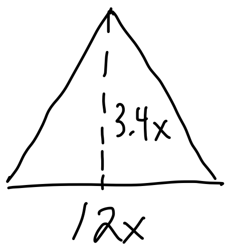
e) $\frac{-14.2m^{\cancel{2}}}{2\cancel{m}}$

$= -7.1m$

 Name: Key

Example 7: Apply monomial multiplication.

A Triangle has a base of $12x$ cm and a height of $3.4x$ cm. What is the area of the triangle?



$$A = \frac{b \times h}{2}$$

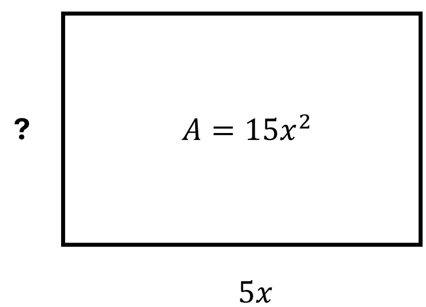
$$A = \frac{(12x)(3.4x)}{2} = \frac{\overset{6}{\cancel{12}} \cdot 3.4 \cdot x \cdot x}{\cancel{2}}$$

$$= 6 \cdot 3.4 \cdot x \cdot x$$

$$= \boxed{20.4x^2 \text{ cm}^2}$$

Example 8: Apply monomial division.

What is the missing dimension of the figure?



$$A = l \times w$$

$$w = \frac{A}{l}$$

$$w = \frac{\cancel{15}x^{\cancel{2}}}{\cancel{5}\cancel{x}}$$

$$w = 3x$$

The missing dimension is 3x.

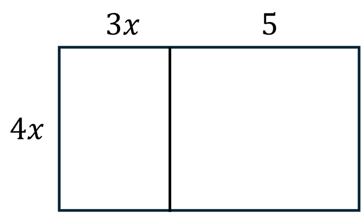
Practice: Textbook pg. 132/# 1 - 7, 10, 12ace, 13ace, 14

Math 9 Name: Key

4.4 – Multiplying Polynomials by Monomials

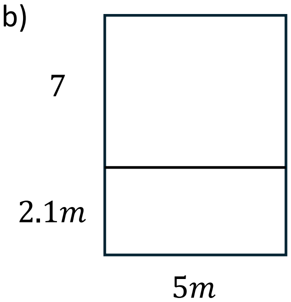
Example 1: What polynomial multiplication statement is represented by each area model?

a)



$(4x)(3x+5)$

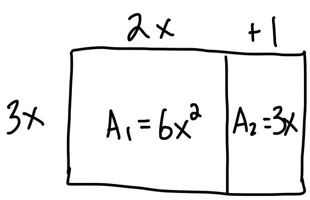
b)



$(5m)(2.1m+7)$

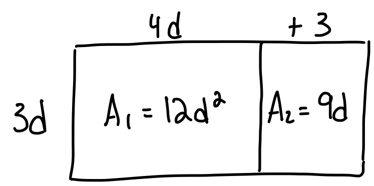
Example 2: Use an area model to determine product.

a) $(3x)(2x+1)$



$A = 6x^2 + 3x$

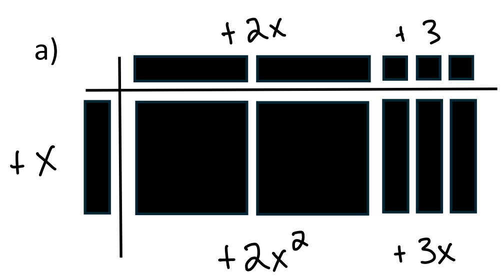
b) $(4d+3)(3d)$



$A = 12d^2 + 9d$

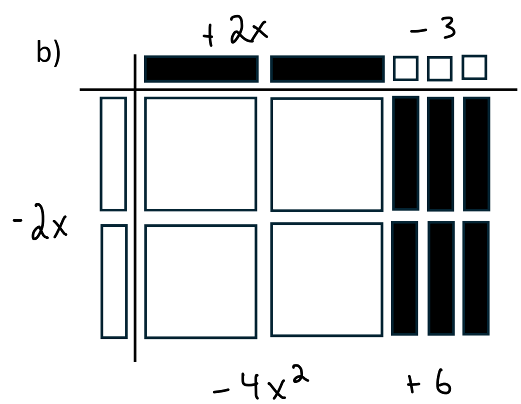
Example 3: Determine the polynomial multiplication statement shown by the algebra tiles.

a)



$(2x+3)(x) = 2x^2 + 3x$

b)

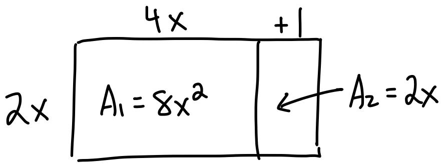


$(-2x)(2x-3) = -4x^2 + 6x$

Math 9 Name: Key

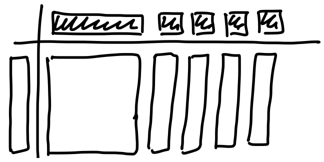
Example 4: Use a model to expand each expression.

a) $(4x + 1)(2x)$



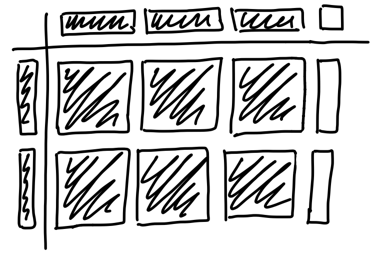
$A = 8x^2 + 2x$

b) $(-x)(x + 4)$



$(-x)(x+4) = -x^2 - 4x$

c) $(2x)(3x - 1)$



$(2x)(3x-1) = 6x^2 - 2x$

Distributive Property: When multiplying a monomial by a polynomial, multiply the Monomial by each term in the polynomial.

$$a(x + y) = ax + ay$$

Example 5: Use the distributive property to multiply polynomials by monomials.

a) $4(3x - 5)$

$= (4)(3x) - (4)(5)$

$= 12x - 20$

b) $-7y(2x - 4y)$

$= (-7y)(2x) - (-7y)(4y)$

$= -14xy + 28y^2$

c) $2x(6x^2 + 3x - 1)$

$= (2x)(6x^2) + (2x)(3x) - (2x)(1)$

$= 12x^3 + 6x^2 - 2x$

Give it a Try: Use the distributive property to multiply the polynomials.

a) $(4m + 1)(3m)$

$= (4m)(3m) + (1)(3m)$

$= 12m^2 + 3m$

b) $(-4x)(2x - 3)$

$(-4x)(2x) - (-4x)(3)$

$= -8x^2 - (-12x)$

$= -8x^2 + 12x$

c) $(\frac{2}{3}m + 4)(-9m)$

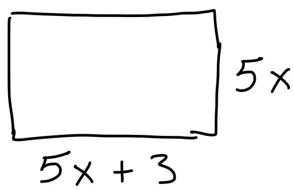
$(\frac{2}{3}m)(-9m) + (4)(-9m)$

$= -6m^2 - 36m$

Math 9 Name: Key

Example 6: The length of a cement pad on a playground in 3m longer than the width. The width is $5x$ m.

a) Write an expression for the area of the cement pad.



$$A = (5x)(5x+3)$$
$$A = 25x^2 + 15x$$

b) If $x = 2$ m, what is the area of the cement pad?

$$A = 25(2)^2 + 15(2)$$
$$= 25(4) + 30$$
$$= 100 + 30$$
$$= 130 \text{ m}^2$$

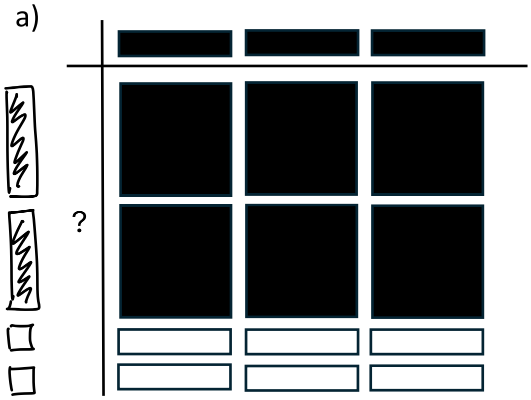
Practice: Textbook pg. 139/# 1-3, (4-6) ace, 8ace, 9ace, 11ace, 13

Math 9 Name: Key

4.5 – Dividing Polynomials by Monomials

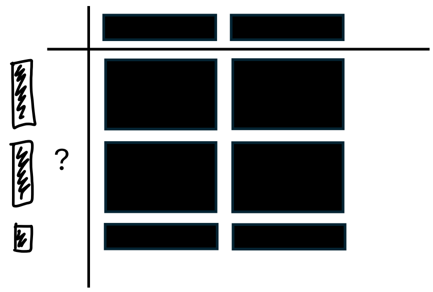
Example 1: What polynomial division statement is represented by the algebra tiles? Determine the quotient.

a)



$$\frac{6x^2 - 6x}{3x} = 2x - 2$$

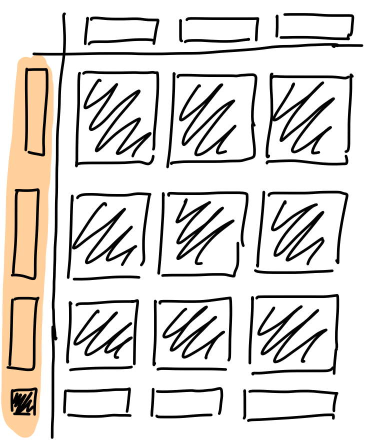
b)



$$\frac{6xy + 2x}{2x} = 2x + 1$$

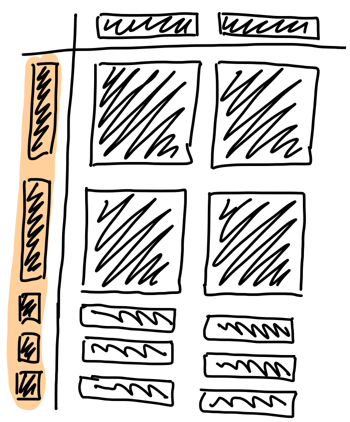
Example 2: Use algebra tiles to divide each of the following expressions.

a) $\frac{9x^2-3x}{-3x}$



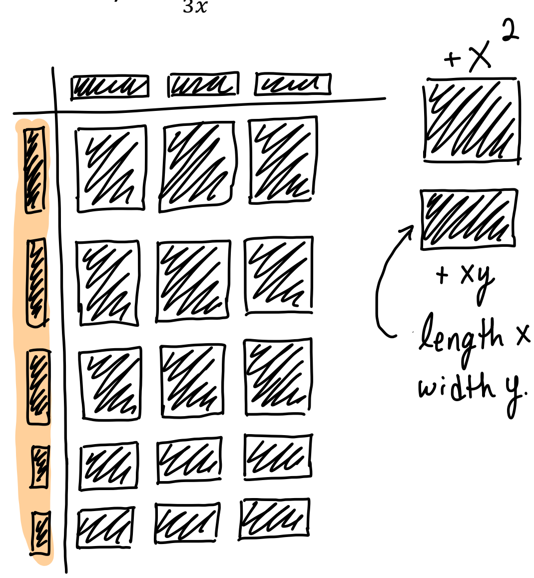
$= -3x + 1$

b) $\frac{4x^2+6x}{2x}$



$= 2x + 3$

c) $\frac{9x^2+6xy}{3x}$



$= 3x + 2y$

 Name: Key

- When dividing a polynomial by a monomial, divide each term in the numerator by the denominator and apply the exponent laws.

Example 3: Divide the following expressions.

a) $\frac{15x^2-20x}{5x}$

$= \frac{\cancel{15}^{3}x^{\cancel{2}}}{\cancel{5}\cancel{x}} - \frac{\cancel{20}^{4}\cancel{x}}{\cancel{5}\cancel{x}}$

$= 3x - 4$

b) $\frac{16m^2+20mn}{4m}$

$= \frac{\cancel{16}^{4}m^{\cancel{2}}}{\cancel{4}\cancel{m}} + \frac{\cancel{20}^{5}\cancel{m}n}{\cancel{4}\cancel{m}}$

$= 4m + 5n$

c) $\frac{18k^2-9k}{9k}$

$= \frac{\cancel{18}^{2}k^{\cancel{2}}}{\cancel{9}\cancel{k}} - \frac{\cancel{9k}}{\cancel{9k}}$

$= 2k - 1$

d) $\frac{12m+18mn}{-6m}$

$= \frac{\cancel{12}^{2}\cancel{m}}{-\cancel{6}\cancel{m}} + \frac{\cancel{18}^{3}\cancel{m}n}{-\cancel{6}\cancel{m}}$

$= -2 - 3n$

e) $\frac{9c^2-12c+6}{-3}$

$= \frac{\cancel{9}^{3}c^2}{-\cancel{3}} - \frac{\cancel{12}^{4}c}{-\cancel{3}} + \frac{\cancel{6}^{2}}{-\cancel{3}}$

$= -3c^2 + 4c - 2$

f) $\frac{1.4d^2+1.8dk-1.6d}{2d}$

$= \frac{1.4d^{\cancel{2}}}{2\cancel{d}} + \frac{1.8\cancel{d}k}{2\cancel{d}} - \frac{1.6\cancel{d}}{2\cancel{d}}$

$= 0.7d + 0.9k - 0.8$

Example 6: You are decorating the bulletin board in your classroom with pictures of your classmates. Each picture covers an area of $4x$ cm^2. The area of the board is $4x^2 + 16x$ cm^2. Write an expression to represent how many pictures are required to cover the board.

Area of board ÷ Area of picture

$\frac{4x^2+16x}{4x}$

$= \frac{\cancel{4}x^{\cancel{2}}}{\cancel{4}\cancel{x}} + \frac{\cancel{16}^{4}\cancel{x}}{\cancel{4}\cancel{x}}$

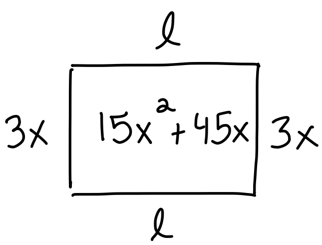
$= x + 4$

$\boxed{x + 4 \text{ pictures are required to cover the board.}}$

Name: Key

Example 7: A rectangular lawn has a width of $3x$ m. The area is $15x^2 + 45x$ m². You wish to put a fence around the lawn.

a) What is an expression to represent the perimeter of the lawn?



$A = lw$

$l = \frac{A}{w}$

$l = \frac{15x^2 + 45x}{3x}$

$= \frac{15x^2}{3x} + \frac{45x}{3x}$

$= 5x + 15$

$P = 2l + 2w$

$P = 2(5x+15) + 2(3x)$

$P = 10x + 30 + 6x$

$\boxed{P = 16x + 30 \text{ m}}$

b) You are placing a post every 2 m. Find an expression to represent how many posts will be required.

$\#\text{ of posts} = \frac{16x + 30}{2}$

$= \frac{16x}{2} + \frac{30}{2}$

$= 8x + 15$

$\boxed{8x+15 \text{ posts will be required.}}$

Practice: Textbook pg. 146/# 1-3, 6-9, 10ab, 12, 13